EE2030010: Linear Algebra **Spring 2018**
Department of Electrical Engineering
National Tsing Hua University

Homework #2
Coverage: chapter 1-3
Due date: 11 April 2018

Instructor: Chong-Yung Chi *TAs: Amin Jalili & Yao-Rong Syu*

Notice:
1. Please hand in the hardcopy of your answer sheets to the TAs by yourself before 23:59 of the due date. No late homework will be accepted.
3. Ansewrs to the problem set should be written on the A4 papers.
4. Write your name, student ID, email and department on the begining of your ansewr sheets.
5. Please justify your answers with clear, logical and solid reasoning or proofs.
6. Please do the homework independently by yourself. However, you may discuss with someone else but copyied homework is not allowed. This will show your respect toward the academic integrity.
7. Your legible handwriting is fine. However, you are very welcome to use text formatting packages for writing your answers.

Problem 1. (20 points) The matrix $\mathbf{A} \in \mathbb{R}^{n\times n}$ can be written as $\mathbf{A} = \mathbf{LDU}$, where $\mathbf{L}$ and $\mathbf{U}$ are lower unitriangular and upper unitriangular matrices, respectively, and $\mathbf{D}$ is a diagonal matrix. Show that LDU decomposition can be reduced to $\mathbf{A} = \mathbf{LDL}^T$ if $\mathbf{A}$ is a nonsingular symmetric matrix.

Problem 2. (20 points) Let V be a finite dimensional vector space and $W_1, W_2 \subset V$ be two subspaces. Then *sum* of two subspaces is defined as

$$S \triangleq \{\mathbf{w}_1 + \mathbf{w}_2 \,|\, \mathbf{w}_1 \in W_1, \mathbf{w}_2 \in W_2\} \subset V,$$

(i) Show that S is a subspace of V.

(ii) Prove that S is the smallest subspace of V which contains W_1 and W_2.

Problem 3. (20 points) Let $\mathbf{A} = \begin{bmatrix} c & c & c & c \\ -c & -c & c & c \\ 0 & 0 & c & c \end{bmatrix}$ where $c \in \mathbb{R}$ and $c \neq 0$. Let's denote $C(.)$, $N(.)$, $rank(.)$ are the column space, null space and rank of the corresponding matrix, respectively.

(i) Find $C(\mathbf{A})$, $C(\mathbf{A}^T)$, $N(\mathbf{A}^T)$, $rank(\mathbf{A}^T)$ and $dim(N(\mathbf{A}^T))$.

(ii) Show that $C(\mathbf{A}) \perp N(\mathbf{A}^T)$.

Problem 4. (20 points) Let P_2 be the the vector space of polynomials with real coefficients and degree less than or equal to 2. Determine whether or not the set of vectors $\{x+1, x^2-3x+1, 2x^2+4\}$ is linearly independent.

Problem 5. (20 points) Let $V = \mathbb{R}^3$. Suppose

$$W_1 = span\left\{ \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 8 \end{bmatrix} \right\}, W_2 = span\left\{ \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\},$$

are subspaces of V. Find a basis for $W_1 \cap W_2$.